## Exercise III: Energy and exergy balance of a Diesel engine

Solutions:

Preliminary computations:

Expression of the Air / Fuel ratio of the mixture  $R_{A/F}$ :

$$\begin{split} \frac{N_A}{M_F} &= \frac{\lambda}{0.21} \cdot \frac{N_{O2,st}}{M_F} = \frac{\lambda}{0.21} \cdot \left( \frac{c_C^F}{\widetilde{m}_C} + 0.5 \cdot \frac{c_{H2}^F}{\widetilde{m}_{H2}} + \frac{c_S^F}{\widetilde{m}_S} - \frac{c_{O2}^F}{\widetilde{m}_{O2}} \right) = 0.5977 \; kmolA/kgF \\ R_{A/F} &= \frac{M_A}{M_F} = \frac{N_A}{M_F} \cdot \widetilde{m}_A = 17.24 \; kgA/kgF \end{split}$$

1) Energy balance of the system (F):  $\dot{E}_e^- = \dot{Y}_{comb}^+ - \dot{Q}_G^- - \dot{Q}_{ae}^- - \dot{Q}_{ab}^- - \dot{Q}_{ae}^-$ 

Heat rate losses of the exhaust gases:  $\dot{Q}_{G}^{-} = \dot{M}_{G} \cdot \hat{h}_{G} + \sum \dot{M}_{ii} \cdot \Delta h_{il}^{0}$ 

with  $\hat{h}_{G} = c_{p,G} \cdot (\hat{T}_{G2} - \hat{T}^{0}) = 630 \text{ kJ/kg}$ 

 $\bar{c}_{p,G}(\lambda = 1.2; \hat{T}_{G2} = 550^{\circ}C) = 1200 \ J/kgK$ 

Conservation of mass (mass balance)  $\dot{M}_G = \dot{M}_A + \dot{M}_E = 0.10707 \ kg/s$ 

 $\dot{M}_A = R_{AF} \cdot \dot{M}_F = 0.1012 \ kg / s$ 

Only the carbon monoxide is unburned, so:  $\sum \dot{M}_{iI} \cdot \Delta h_{iI}^0 = \dot{M}_{CO} \cdot \Delta h_{CO}^0$ 

 $\dot{Q}_G^- = 0.10707 \cdot 630 + 0.000258 \cdot 10100 = 70.1 \, kW$ 

Heat rate losses of the cooling water network:  $\dot{Q}_{qe}^- = \dot{M}_e \cdot c_{pe} \cdot (\hat{T}_{e2} - \hat{T}_{e1})$ 

 $\dot{Q}_{ae}^{-} = 50.8 \ kW$ 

Heat rate losses of the lubricating oil network:  $\dot{Q}_{ah}^- = \dot{M}_h \cdot c_{p,h} \cdot (\hat{T}_{h2} - \hat{T}_{h1})$ 

 $\dot{Q}_{ah}^{-} = 6.9 \; kW$ 

Fuel transformation power:  $\dot{Y}_{comb}^{+} = \dot{M}_{F} \cdot \Delta h_{i}^{0} + \hat{Y}_{i}^{+}$ 

where  $\,\hat{\dot{Y}}_{i}^{+}$  is the relative fuel transformation power given by:

$$\hat{\dot{Y}}_{i}^{+} = \dot{M}_{cond} \cdot q_{vap}^{0} + \left(\dot{M}_{F} \cdot \hat{h}_{F} + \dot{M}_{A} \cdot \hat{h}_{A} - \dot{M}_{G} \cdot \hat{h}_{G}\right)$$

As atmospheric state is equal to standard state, the second term of the right member is equal to:

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$$(\dot{M}_F \cdot \hat{h}_F + \dot{M}_A \cdot \hat{h}_A - \dot{M}_G \cdot \hat{h}_G) = 0$$

Then, considering the hypothesis, we have finally:  $\hat{\dot{Y}}_i^+ = \dot{M}_{cond} \cdot q_{van}^0 = 0$ 

So finally, transformation power is equal to:

$$\dot{Y}_{comb}^{+} = \dot{M}_{F} \cdot \Delta h_{i}^{0}$$

$$\dot{Y}_{comb}^{+} = 250 \ kW$$

Heat rate dependition by convection / radiation:

$$\dot{Q}_{ac}^{-} = \dot{Y}_{comb}^{+} - \dot{Q}_{G}^{-} - \dot{Q}_{ae}^{-} - \dot{Q}_{ah}^{-} - \dot{E}_{e}^{-}$$

$$\dot{Q}_{ac}^{-} = 22.3 \ kW$$

Finally, the energy balance computation is equal to:

Terms:	Output	Input
Mechanical power	$\dot{E}_e^- = 100 \; kW$	$\dot{Y}_{comb}^{+} = 250 \; kW$
Heat rate losses of the exhaust gases	$\dot{Q}_G^- = 70.1  kW$	
Heat rate losses of the cooling water	$\dot{Q}_{ae}^{-} = 50.8 \; kW$	
Heat rate losses of the lubricating oil	$\dot{Q}_{ah}^{-}=6.9~kW$	
Heat rate deperdition due to conv. / rad.	$\dot{Q}_{ac}^{-} = 22.3 \; kW$	
Total	$Total = 250 \ kW$	$Total = 250 \ kW$

2) Energy balance of the system (Boundary F):

$$\dot{E}_{e}^{-} = \dot{Y}_{comb}^{+} - \dot{Q}_{G}^{-} - \dot{Q}_{ae}^{-} - \dot{Q}_{ah}^{-} - \dot{Q}_{ac}^{-}$$

Effectiveness of the system (Boundary F):

$$\varepsilon = \frac{\dot{E}_{e}^{-}}{\dot{Y}_{comb}^{+}} = 1 - \frac{\dot{Q}_{G}^{-} + \dot{Q}_{ae}^{-} + \dot{Q}_{ah}^{-} + \dot{Q}_{ac}^{-}}{\dot{M}_{F} \cdot \Delta h_{i}^{0}}$$

Finally, the global effectiveness of the system F is equal to:

$$\varepsilon = 40 \%$$

Optional:

Exergy balance of the system (Boundary F):

$$\dot{E}_e^- = \dot{E}_{v,comb}^+ - \dot{L}$$

with

$$\dot{E}_{v,comb}^{+} = \dot{M}_{F} \cdot \Delta k^{0}$$

Exergy efficiency of the system (Boundary F):

$$\eta = \frac{\dot{E}_e^-}{\dot{M}_F \cdot \Delta k^0} = 1 - \frac{\dot{L}}{\dot{M}_F \cdot \Delta k^0}$$

Which gives a exergy efficiency of:

$$\eta = 37.9 \%$$

Energy balance of the engine (Boundary F<sub>M</sub>): 
$$\dot{E}_e^- + \dot{Y}_{ae}^- + \dot{Y}_{ah}^- = \dot{Y}_{combM}^+ - \dot{Q}_{ac}^-$$

Transformation power of the cooling water network: 
$$\dot{Y}_{ae}^- = \dot{Q}_{ae}^-$$

$$\dot{Y}_{ae}^{-} = \dot{M}_{e} \cdot c_{p,e} \cdot (\hat{T}_{e2} - \hat{T}_{e1}) = 50.8 \ kW$$

Transformation power of the lubricating water network:  $\dot{Y}^{-}_{ah}=\dot{Q}^{-}_{ah}$ 

$$\dot{Y}_{ah}^{-} = \dot{M}_h \cdot c_{p,h} \cdot (\hat{T}_{h2} - \hat{T}_{h1}) = 6.9 \ kW$$

## Fuel transformation power:

$$\dot{Y}_{combM}^{+} = \dot{M}_{F} \cdot \Delta h_{i}^{0} - \sum \dot{M}_{iI} \cdot \Delta h_{iI}^{0} + \dot{\hat{Y}}_{M}^{+}$$

where  $\hat{\hat{Y}}_{\!\scriptscriptstyle M}^{\scriptscriptstyle +}$  is the relative fuel transformation power of the subsystem F given by (see question 2):

$$\hat{\dot{Y}}_{M}^{+} = \dot{M}_{cond} \cdot q_{vap}^{0} + \left(\dot{M}_{F} \cdot \hat{h}_{F} + \dot{M}_{A} \cdot \hat{h}_{A} - \dot{M}_{G} \cdot \hat{h}_{G}\right)$$

As atmospheric state is equal to standard state and condensation water flow rate is neglected.

Therefore: 
$$\hat{\dot{Y}}_{M}^{+} = -\dot{M}_{G} \cdot \hat{h}$$

and finally: 
$$\dot{Y}_{combM}^{+}=\dot{M}_{F}\cdot\Delta h_{i}^{0}-\dot{M}_{CO}\cdot\Delta h_{CO}^{0}-\dot{M}_{G}\cdot\hat{h}_{G}$$

$$\dot{Y}_{combM}^{+} = 180 \ kW$$

Heat rate dependition by convection / radiation:  $\dot{Q}_{ac}^- = \dot{Y}_{combM}^+ - \dot{Y}_{ae}^- - \dot{Y}_{ah}^- - \dot{E}_e^-$ 

$$\dot{Q}_{ac}^{-} = 22.3 \; kW$$

But, keep in mind that a term of the Fuel transformation power is a potential of energy service provided by the system. Therefore, we also have:

Transformation power of the exhaust gases:  $\dot{Y}_G^- = \dot{M}_G \cdot \hat{h}_G = 67.5 \; kW$ 

Effectiveness of the engine (Boundary  $F_M$ ):

$$\varepsilon_{M} = \frac{\dot{E}_{e}^{-} + \dot{Y}_{G}^{-} + \dot{Y}_{ae}^{-} + \dot{Y}_{ah}^{-}}{\dot{M}_{E} \cdot \Delta h_{e}^{0} - \dot{M}_{CO} \cdot \Delta h_{CO}^{0}} = \frac{100 + 67.5 + 50.8 + 6.9}{250 - 2.6}$$

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Finally, the global effectiveness of the system  $F_{\rm M}$  is equal to:  $\varepsilon_{\rm M}=91.0~\%$ 

## Optional:

Exergy balance of the engine (Boundary F<sub>M</sub>): 
$$\dot{E}_e^- + \dot{E}_{vae}^- + \dot{E}_{vah}^- = \dot{E}_{v.combM}^+ - \dot{L}_M$$

Considering the following relations: 
$$\dot{E}_{\nu,combM}^{+} = \dot{M}_{F} \cdot \Delta k^{0} - \dot{M}_{CO} \cdot \Delta k_{CO}^{0} - \dot{M}_{G} \cdot \Delta \hat{k}_{G2}$$

$$\dot{E}_{vae}^{-} = \dot{M}_{e} \cdot (k_{e2} - k_{e1})$$

$$\dot{E}_{yah}^{-} = \dot{M}_{e} \cdot (k_{h2} - k_{h1})$$

## Exergy transformation of the exhaust gases:

Calculation of exergy value of the exhaust gases:

$$\hat{k}_{G2} = \hat{h}_G - T_a \hat{s}_G = \hat{h}_G - T_a \cdot c_G \cdot \ln\left(\frac{T_G}{T_a}\right) = 266.7 \text{ kJ/kg}$$

Finally, 
$$\dot{M}_{\scriptscriptstyle G}\cdot \hat{k}_{\scriptscriptstyle G2}=28.6~kW$$

Fuel exergy transformation:

$$\dot{E}_{v,combM}^{+} = \dot{M}_{F} \cdot \Delta k^{0} - \dot{M}_{CO} \cdot \Delta k_{CO}^{0} - \dot{M}_{G} \cdot \Delta \hat{k}_{G2} = 233 \text{ kW}$$

Exergy transformation of the water cooling network:

$$\dot{E}_{yae}^{-}$$

Calculation of exergy value of the water cooling network:

$$\begin{split} \hat{k}_{e1} &= \hat{h}_{e1} - T_a \hat{s}_{e1} = c_e \cdot \left(\hat{T}_{e1} - \hat{T}^0\right) - T_a \cdot c_e \cdot \ln\left(\frac{T_{e1}}{T_a}\right) = 22.4 \; kJ \, / \, kg \\ \\ \hat{k}_{e2} &= \hat{h}_{e2} - T_a \hat{s}_{e2} = c_e \cdot \left(\hat{T}_{e2} - \hat{T}^0\right) - T_a \cdot c_e \cdot \ln\left(\frac{T_{e2}}{T_a}\right) = 26.04 \; kJ \, / \, kg \end{split}$$
 Finally, 
$$\dot{E}_{vae}^- = 8.8 \; kW$$

Exergy transformation of the lubricating oil network:  $\dot{E}_{yah}^-$ 

Calculation of exergy value of the lubricating oil network:

$$\begin{split} \hat{k}_{h1} &= \hat{h}_{h1} - T_a \hat{s}_{h1} = c_h \cdot \left( \hat{T}_{h1} - \hat{T}^0 \right) - T_a \cdot c_h \cdot \ln \! \left( \frac{T_{h1}}{T_a} \right) = 11.16 \; kJ / \, kg \\ \hat{k}_{h2} &= \hat{h}_{h2} - T_a \hat{s}_{h2} = c_h \cdot \left( \hat{T}_{h2} - \hat{T}^0 \right) - T_a \cdot c_h \cdot \ln \! \left( \frac{T_{h2}}{T_a} \right) = 14.58 \; kJ / \, kg \end{split}$$
 Finally, 
$$\dot{E}_{vah}^- = 1.30 \; kW$$

Exergy loss (according exergy balance):

$$\dot{L}_{M} = \dot{E}_{y,combM}^{+} - \dot{E}_{e}^{-} - \dot{E}_{yae}^{-} - \dot{E}_{yah}^{-}$$

$$\dot{L}_{M} = 233 - 100 - 8.8 - 1.3 = 122.9 \ kW$$

Exergy efficiency of the engine (Boundary  $F_M$ ):

$$\eta_{M} = \frac{\dot{E}_{e}^{-} + \dot{M}_{G} \cdot \hat{k}_{G2} + \dot{E}_{yae}^{-} + \dot{E}_{yah}^{-}}{\dot{M}_{F} \cdot \Delta k^{0} - \dot{M}_{CO} \cdot \Delta k_{CO}^{0}}$$

Numerical application:

$$\eta_M = \frac{100 + 28.6 + 8.8 + 1.3}{264.1 - 2.54}$$

Which gives an exergy efficiency:

$$\eta_{M} = 53.0 \%$$

4) If the unburned hydrocarbons are considered as part of the energy losses, the engine effectiveness is:

$$\varepsilon_{M}^{'} = \frac{\dot{E}_{e}^{-} + \dot{Y}_{G}^{-} + \dot{Y}_{ae}^{-} + \dot{Y}_{ah}^{-}}{\dot{M}_{F} \cdot \Delta h_{i}^{0}} = 90.0 \%$$

Optional:

And the exergy efficiency is equal to:

$$\eta_{M}^{'} = \frac{\dot{E}_{e}^{-} + \dot{M}_{G} \cdot \hat{k}_{G2} + \dot{E}_{yae}^{-} + \dot{E}_{yah}^{-}}{\dot{M}_{F} \cdot \Delta k^{0}} = 52.5 \%$$